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Pure Bound Field theory and the Decay of Moun in Meso-Atoms

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Abstract In the present paper we consider the electrically bound quantum particles within the framework of pure bound field theory (PBFT) (Kholmetskii, A.L., et al.: *Phys. Scr.* **82**, 045301 (2010)), which explicitly takes into account the non-radiative nature of electromagnetic (EM) field generated by bound charges in the stationary energy states, and evokes the appropriate modifications of bound EM field, which secure the total momentum conservation law for the isolated system "electron plus nucleus" in the absence of EM radiation. Such a PBFT gives the same gross as well as fine structure of atomic energy levels, as those furnished by the common approach, but implies a scaling transformation of radial coordinates. In this paper we find out that in the classical limit this transformation reflects the dependence of time rate for the orbiting electron on the electric potential of the binding EM field in addition to relativistic dependence on its Lorentz factor. We show that this effect completely eliminates the available up to date discrepancy between calculated and experimental data on the decay rate of bound muon in meso-atoms. We emphasize that the revealed dependence of time rate of quantum electrically bound particles on the electric potential represents the specific effect of PBFT, and, in general, is not extended to the classical world.

Keywords Meso-atoms · Decay of bound muon · Pure bound field theory

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1 Introduction

In our recent paper [1] we pointed out the known fact that in quantum physics electrically bound particles do not radiate in stationary energy states and thus their electromagnetic (EM) field consists of the bound (velocity-dependent) component only. This effect does not have a classical analogy, where, as known, an orbiting charge must inevitably radiate, and both bound and radiative EM field components equally participate in securing the total momentum conservation law. Hence the question emerges, which appears to have not been asked before Ref. [1]: How does Nature restore the momentum conservation law for quantum *bound* systems of charges, where their EM radiation is prohibited?

From the general viewpoint, it is clear that a possible way to implement the momentum conservation law for an isolated system of charges with the prohibited EM radiation is to modify in an appropriate way their bound EM field (potentials) and/or the relationships between the fields (potentials) and their energy-momentum. Exploring this problem and considering the electron in an external EM field, we suggested the appropriate modification of the Dirac equation for bound electron (i.e., when its total energy $E < mc^2$, m being electron's rest mass and c the light velocity in vacuum), which explicitly takes into account the non-radiative nature of its EM field [1]. The introduction of corresponding corrections into the Dirac-Coulomb equation [1] and later into the Breit equation without external field [2] gave rise to the development of Pure Bound Field quantum Theory (PBFT), which we proposed. When applied to the hydrogenlike atoms, PBFT yields the same gross and fine structure of atomic energy levels, as those furnished by the common approach [1], but brings some important corrections at the scale of hyperfine interactions, which amazingly eliminate all the available discrepancies between theory and experiment in physics of light hydrogen-like atoms ($1S$–$2S$ interval in positronium, hyperfine spin-spin splitting in positronium, $2S$–$2P$ Lamb shift and $1S$ Lamb shift in hydrogen [2]). In particular, the proton charge radius re-estimated in PBFT from the classic Lamb shift comes to be equal to 0.841(6) fm, which perfectly agrees with the latest experimental result 0.84184(67) fm [3].

In Sect. 2, for the convenience of the reader, we shortly reproduce from [1] the solution of the modified Dirac-Coulomb equation up to the order $(Z\alpha)^4$, where Z is the atomic number and α the fine structure constant. As we have mentioned above, this solution coincides with the common solution for the atomic energy levels, but concurrently implies a scaling transformation of radial coordinates. The latter was interpreted in [1] as the change of atomic form-factors, which however cannot be observed at the present measuring precision.

In the present contribution (Sect. 3) we propose an alternative physical interpretation of this scaling transformation and, based on the general relativistic analysis, suggest to consider it as the consequence of dilation of time for the bound electron as the function of electric potential of the nucleus. Such a phenomenon could be classified as a specific effect of PBFT in quantum physics of electrically bound systems, which, in general, is not extended to classical phenomena. A verification of this effect can be done with meso-atoms, where the negative muon being captured by the atom possesses a property to directly exhibit its time rate via the decay rate τ_b. Since the strength of electric field E and the electric potential φ are linearly proportional to the atomic number Z, we get a unique possibility to observe the dependence $\tau_b(\varphi)$, measuring the decay rate of bound muons in various atoms. The analysis of experimental data implemented in Sect. 3 fully supports the predicted phenomena. Finally, we conclude in Sect. 4.

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2 Pure Bound Field Theory and the Structure of Atomic Energy Levels

We remind that the Dirac equation (DE) written for the electron e in some external electromagnetic (EM) field with the four-potential $A_i\{\boldsymbol{A},\varphi\}$, has the form (e.g., [4])

$$i\hbar\frac{\partial\psi'}{\partial \mathrm{t}}=\left[\mathrm{c}\alpha\left(\hat{\mathbf{P}}-\frac{\mathrm{e}}{\mathrm{c}}\mathbf{A}\right)+\beta \mathrm{mc}^2+\mathrm{e}\varphi\right]\psi', \tag{1}$$

where $\psi'=\binom{\xi}{\chi}$ is the wave function, m is the electron's mass, c the light velocity in vacuum, $\alpha=\begin{pmatrix}0&\sigma\\ \sigma&0\end{pmatrix}$, $\beta=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $\hat{\mathbf{P}}_b=-i\hbar\nabla$ is the operator of momentum, and $\boldsymbol{\sigma}$ is the Pauli matrix. Herein the term $e\varphi$ represents the electric interaction (potential) energy of electron, and the term $\hat{\mathbf{P}}$ stands for the canonical momentum of electron in an external EM field.

In order to explicitly take into account the non-radiative nature of EM field for the bound electron, and requiring the legality of the total momentum conservation law in the absence of EM radiation, we suggest in Ref. [1] modifying (1) as

$$\left(i\hbar\frac{\partial}{\partial t}+mB_nc^2\right)\psi'=\left[c\alpha\left(\hat{\mathbf{P}}_b-\frac{e}{c}\mathbf{A}_b\right)+B_n\beta mc^2+\Gamma_n e\varphi\right]\psi', \tag{2}$$

where B_n, Γ_n are the step-wise functions defined by the relationships

$$B_n\left(E-mc^2\right)=\begin{matrix}1 & (at\,E\ge mc^2),\\ b_n & (at\,E<mc^2),\end{matrix}\qquad \Gamma_n\left(E-mc^2\right)=\begin{matrix}1 & (at\,E\ge mc^2),\\ \gamma_n & (at\,E<mc^2),\end{matrix} \tag{3}$$

where b_n, γ_n are coefficients, we introduce. These coefficients are aimed to reflect the non-radiative nature of EM field of bound electron within the total momentum conservation constraint and in the non-relativistic limit both b_n and γ_n are put to be equal to unity. Hence their introduction in the Dirac equation does not affect the gross structure of the energy levels for bound electron, characterized by the principal quantum number n. As shown in [1], the coefficients b_n, γ_n depend only on quantum numbers, providing thus the Lorentz-invariance of (2) with the step-wise function (3), when the non-relativistic limit is no longer assumed [1].

The definition (3) is made by such a way, where for $E\ge mc^2$(free electron), (2) takes the common form (1). From the physical viewpoint, the presence of step-wise functions (3) in the structure of Dirac equation reflects a discontinuity of the properties of the electron, which can emit EM radiation at the total energy $E\ge mc^2$, but loses the ability to radiate at $E<mc^2$.

Next we have to postulate an explicit reasonable form for the coefficients b_n and γ_n in (3). For this purpose we considered in [1] a model classical one-body problem, where EM radiation of the electron orbiting around heavy nucleus is prohibited, and further determined the way to modify the bound EM field of this system, which provides the implementation of total momentum conservation law. On this way we derived the classical limits of the coefficients b_n, γ_n as follows:

$$b=1+e\varphi/mc^2, \tag{4a}$$

$$\gamma=\left(1-v^2/c^2\right)^{-1/2}, \tag{4b}$$

where v is the orbital velocity of the classical electron and γ represents its Lorentz factor.

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Equations (4a)–(4b) disclose an appropriate way for postulating the coefficients b_n and γ_n in (1), so that the values b and γ should represent their classical limit.

Having obtained the latter result, we further modified the DE for bound electron into Dirac–Coulomb equation for the quantum one-body problem [1]

$$\left(\frac{\hat{\mathbf{P}}_b{}^2}{2mb_n} - \gamma_n\frac{Ze^2}{r} - \frac{\hat{\mathbf{P}}_b{}^4}{8m^3b_n^3c^2} - \frac{e\hbar\hat{\mathbf{s}}\cdot(\mathbf{E}_b\times\hat{\mathbf{P}}_b)}{2m^2b_n^2c^2} - \frac{e\hbar^2(\nabla\cdot\mathbf{E}_b)}{8m^2b_n^2c^2}\right)\psi(\mathbf{r},\vartheta,\varphi) = W\psi(\mathbf{r},\vartheta,\varphi) \tag{5}$$

in the designations of Ref. [1]. In particular, the electric field $\mathbf{E}_b = \gamma_n\mathbf{E} = \gamma_n Ze\mathbf{r}/r^3$, the operator of effective momentum $\hat{\mathbf{P}}_b = -i\hbar\nabla$ in the classical limit is determined by the expression

$$\mathbf{P} = \gamma bm\mathbf{v}, \tag{6a}$$

where the parameter

$$m_b = b_n m \tag{6b}$$

can be named as the *effective rest mass* of the bound electron. Further, $\hat{\mathbf{s}}$ is the spin operator, $\psi(\mathbf{r},\vartheta,\varphi)$ is the wave function, and W is the energy.

Via the substitution [1]

$$\mathbf{r} = \mathbf{r}'/(b_n\gamma_n). \tag{7}$$

Equation (5) can be transformed to the convenient form

$$\left[\hat{H}_0(\mathbf{r}') + \hat{V}(\mathbf{r}')\right]\psi(\mathbf{r}',\vartheta,\varphi) = W'\psi(\mathbf{r}',\vartheta,\varphi), \tag{8}$$

where

$$\hat{H}_0(\mathbf{r}') = -\frac{\hbar^2\nabla^2_{\mathbf{r}'}}{2m} - \frac{Ze^2}{r'}$$

is the conventional non-relativistic Schrödinger operator, expressed via $\mathbf{r}'$ coordinates (so that $\nabla_{r'} = \partial/\partial\mathbf{r}'$), whereas

$$\hat{V}(\mathbf{r}') = \gamma_n^2\left(-\frac{\hat{\mathbf{P}}_b{}^4(\mathbf{r}')}{8m^3c^2} - \frac{e\hbar\hat{\mathbf{s}}\cdot(\mathbf{E}(\mathbf{r}')\times\hat{\mathbf{P}}_b(\mathbf{r}'))}{2m^2c^2} - \frac{e\hbar^2(\nabla_{\mathbf{r}'}\cdot\mathbf{E}(\mathbf{r}'))}{8m^2c^2}\right) \tag{9}$$

is the perturbation term. We also designated $W' = W/b_n\gamma_n^2$.

Thus, applying the perturbation theory, we first observe that the standard Schrödinger equation

$$\left(-\frac{\hbar^2\nabla^2_{\mathbf{r}'}}{2m} - \frac{Ze^2}{r'}\right)\psi\left(\mathbf{r}',\vartheta,\varphi\right) = W'\psi\left(\mathbf{r}',\vartheta,\varphi\right)$$

gives the known solution for the stationary energy levels W_n as well as the standard Schrödinger–Coulomb wave function $\psi_n(\mathbf{r}',\vartheta,\varphi)$ for the hydrogenlike atom. Hence we can apply the known relationships [5]:

$$\overline{v_n^2} = (Z\alpha)^2c^2/n^2, \qquad 1/\overline{r_n'} = (Z\alpha/n^2)mc/\hbar$$

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and determine the parameters b_n, γ_n as follows, based on their classical counterparts (4a)–(4b) [1]:

$$b_n = 1 - (Z\alpha)^2/n^2, \tag{10a}$$

$$\gamma_n = (1 - (Z\alpha)^2/n^2)^{-1/2} \tag{10b}$$

for each stationary energy state n. Hence we obtain

$$b_n \gamma_n^2 = 1, \tag{11}$$

at least to the order $(Z\alpha)^2$. Taking also into account that the perturbation term (9) itself has the order of magnitude $(Z\alpha)^4$, we conclude that the corrections of PBFT to this term, expressed via the coefficient γ_n^2 (10b), appear not before the order $(Z\alpha)^6$. Hence it is seen that (8) yields the same gross and fine structure for the atomic energy levels of light hydrogenic atoms, like in the common approach.

One can show that for the quantum one-body problem the solution of (8) for the atomic energy levels coincides with the corresponding solution of common Dirac–Coulomb equation[1] up to the order $(Z\alpha)^6$ [2].

Now we focus our attention to the physical meaning of the scale transformation (7), which was interpreted in Ref. [1] as the change of atomic form-factor by $b_n \gamma_n$ times. Based on the obtained expressions for the coefficients b_n and γ_n ((10a)–(10b)), one can see that the relative change of the radial coordinate has the order of magnitude $(Z\alpha)^2$ and indeed is well below the present measurement precision (about 1%) of the atomic form-factors.

At the same time, the proposed in Ref. [1] interpretation of the transformation (7) leaves some freedom for the physical status of the radial coordinate $\mathbf{r}$ entering into the original Dirac-Coulomb equation (5). From this standpoint, it is natural, just like the common approach frames, to adopt $\mathbf{r}$ as the laboratory coordinate of electron, so that no changes of form-factors take place in PBFT. However, in such a case the question one may raise about the physical meaning of the transformation (7) remains unanswered.

In order to clarify this problem, it is convenient to return to the classical limit of one-body problem, where we can attach the co-moving Lorentz frames with both the nucleus (K_N) and with the electron (K_e). In the one-body problem, where the mass of the nucleus is supposed to be infinite, the frame K_N coincides with the laboratory frame, where the radial coordinate of electron $\mathbf{r}$ is measured.

However, for the observer in the rotating frame K_e the distance between the electron and the nucleus is not equal to r, if it is measured by means of a light signal exchange between the particles. Indeed, due to the dilation of time in the frame K_e in comparison with K_N,

$$dt_e = \frac{1}{k} dt \tag{12}$$

(where, for more generality, we still prefer to use some coefficient k instead of the Lorentz factor), the total time of light signal exchange in K_e-frame $\Delta t_e = \frac{2r_e}{c} = \frac{2r}{ck}$, and

$$r_e = r/k. \tag{13a}$$

[1] For the quantum two-body problems, the corrections of PBFT to the common solutions have the order of magnitude $(Z\alpha)^6(m/M)$, and along with the radiative corrections to the atomic energy levels, provide much better agreement between theoretical and experimental data in physics of light hydrogenlike atoms [2].

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Herein we also have used the obvious fact that the electric field does not affect the metric of empty space-time and thus the light velocity is equal to c in both K_N and K_e frames.[2] Since (13a) involves only the radial coordinates of the laboratory frame and the frame of electron, it can be directly generalized to the vectorial form

$$\mathbf{r}_e = \mathbf{r}/k. \tag{13b}$$

Thus, comparing now (13b) and (7), one can assume that in the classical limit the $\mathbf{r}'$-coordinate of (7) belongs to the electron's frame, while the coefficient k in (13b) is equal to

$$k = 1/b_n \gamma_n. \tag{14}$$

Such an interpretation of (7) implies that the time rate of the orbiting electron is defined by the expression (sf. (12) and (14))

$$dt_e = b_n \gamma_n dt. \tag{15}$$

In the classical limit this equation acquires the form (see (4a)–(4b))

$$dt_e = b\gamma dt = \frac{dt(1 + e\varphi/mc^2)}{\sqrt{1 - v^2/c^2}}, \tag{16}$$

which predicts a novel physical phenomenon: the change of time rate for the bound charged particle as the function of electric potential of the binding electric field. As it is seen from (16), this effect is not mixed with the conventional relativistic dilation of time due to a motion, and exists even for the resting electron ($v = 0$) in an external electric field:

$$dt_e = bdt = dt(1 + e\varphi/mc^2). \tag{17}$$

One can add that the transformation for the time rate (16) is in a full harmony with the relationship between the effective mass of the bound electron and its proper rest mass ($m_{ef} = b\gamma m$, (6)) and its total energy ($W_{\text{total}} = m_{ef}c^2 = b\gamma mc^2$) within the framework of PBFT. In fact, we observe that the relationship between the time rate, total mass and total energy coincides with the usual relativistic relation between these parameters, when we replace the rest mass m by the effective rest mass bm. Such conformity in transformation of the mentioned parameters (time–mass–energy) takes place only in PBFT due to the appropriate modification of bound EM field in the absence of radiation and, in general, is not extent to the common classical electrodynamics. Thus, as minimum, we have to restrict the range of

[2] One should remind that the metric of three-space in the cylindrical coordinates (r, z, φ) of rotating frame is determined by the equation [6]

$$dl^2 = dr^2 + dz^2 + \frac{r^2 d\varphi^2}{1 - \Omega^2 r^2/c^2},$$

where Ω is the rotation frequency. This equation implies that the radial coordinates in the laboratory and in the rotating frame coincide with each other, in a seeming contradiction with (13). In this connection one should remind that the measurement methods used for determination of metric relationships must obey the requirements of definiteness, reversibility and transitivity [7], whereas the measurement of distances by means of light signal exchange does not meet the requirement of transitivity in a rotating frame [8]. Therefore, (13) does not express the metric relation between the coordinates r_e and r_N, but rather indicates on the different time rate in the frames K_e and K_N.

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validity of (15) by electrically bound quantum systems, which lose the ability of radiating at the stationary energy states.

The analysis we performed above was carried out in the approximation of quantum one-body problem. Nevertheless the effect predicted, i.e. the change of time rate for bound quantum particle versus an electric potential can be easily extended to the two-body problem. This means that both bound charges in stationary energy states have to exhibit a dilation of time (next to the usual Lorentz dilation), as a function of electric potential strength; each particle should thus retard, as much as the ratio of its mass to the total mass of the two. For the atomic nucleus with the finite rest mass M, the change of its time rate in the electric potential of electron has the order of magnitude

$$\frac{e\varphi}{Mc^2} \approx (Z\alpha)^2 \frac{m}{M},$$

and is, in effect, much less than the same effect for the light charge m. For example, for the proton ($M/m = 1836$), the factor $(Z\alpha)^2 m/M \approx 3 \times 10^{-8}$. Another problem is that the stable proton does not give us any "marker" on its time rate and thus we have to look for such bound states, which exhibit a decay.

For muonium (where we have $M/m \approx 207$ and the proper decay rate of free muon $\tau_0 = 2.2 \times 10^{-6}$ s), the factor$(Z\alpha)^2 m/M \approx 3 \times 10^{-7}$, and the corresponding change of half life for muon in muonium

$$\delta\tau \approx 3 \times 10^{-7} \tau_0 \approx 6 \times 10^{-13} \text{ s},$$

which is hardy possible to measure.

For positronium ($m = M$), the factor$(Z\alpha)^2 m/M \approx 5 \times 10^{-5}$, which is less about one order of magnitude than the present relative uncertainty in the measurement of its annihilation rate [9].

As we show in Sect. 4, (15) can be subjected to the experimental test via the measurement of the decay rate of bound muon in meso-atoms.

3 Change of Time Rate for Quantum Electrically Bound Particles: Decay Rate of Muon in Meso-Atoms

The convenient objects for verification of (15) are meso-atoms with different atomic number Z, where the negative muons being captured by the atoms possess a property to directly exhibit their time rate via the decay rate τ_b. Since the strength of electric field E and the electric potential are linearly proportional to the atomic number Z, we get a unique possibility to observe the dependence $\tau_b(\varphi)$, measuring the decay rate of bound muons in various atoms.

The experiments for measurement of the decay rate of muons bound in meso-atoms at various Z had been carried out in 1960's, the last century [10, 11] and they contradict each other in some points. Nonetheless they were not repeated later and thus, we hope that the present analysis will stimulate further research on this subject.

First we remind that a dominant channel of a decay of the free muon is

$$\mu^- \to e^- + \breve{\nu}_e + \nu_\mu \tag{18}$$

with the rate $\tau_0 = 2.2 \times 10^{-6}$ s in its proper reference frame.

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Negative muons being captured by atoms, reach the $1S$ state. Such a muon disappears by two competing processes: nuclear capture and decay (18). The cross-section of nuclear capture rapidly increases with Z; however, using the "start–stop" technique with registration of electrons in the reaction (18), one can measure the rate of this reaction separately.

To the moment, there were known three effects which make the rate of (18) different for bound and free muons: phase space effect, the electron Coulomb effect and relativistic dilation of time [12]. For light atoms, the first and second effects almost eliminate each other, and the relativistic dilation of time prevails. For heavy atoms the situation becomes more complicated. One should notice that the relativistic dilation of time for bound particles in quantum mechanics is not, in general, a trivial effect, because it is impossible to attach an inertial rest frame to such bound particle in the classical meaning. Nonetheless, the relativistic effects to be proportional to the averaged squared velocity of particle (electron, muon) actually appear in the solution of equations of atomic physics (see, e.g. [13]). Thus it is natural to adopt that the time dilation effect for the bound muon is also determined by its squared velocity averaged with the wave function of $1S$ state [12]. The author of [12] made careful numerical calculations of bound muon decay rate τ_b versus Z for all effects mentioned above and plotted the corresponding resulting curve to be shown in Fig. 1. In the same figure we show the experimental results [10], which drastically deviate from the theoretical curve at large Z. In order to explain this deviation, Huff paid attention to the substantial difference between the electron spectra for bound muon decay and that for free muon decay [12]. Hence the decay rate ratio τ_b/τ_0, before comparing with the experimental results, must be corrected for two effects: (1) the energy threshold for detection of the decay electrons; (2) the energy loss by the decay electrons in the target. By this way Huff has corrected the computed value of τ_b/τ_0 for iron ($Z=26$), antimony ($Z=51$), tantalum ($Z=73$) and lead ($Z=82$), which are depicted in Fig. 1 by circles. Nonetheless, the deviation between experimental data and corrected theoretical values still remains appreciable.

Now we are in the position to apply our (15) to the correction of Huff's theoretical data. Here one should take into account that his data already include the relativistic dilation of time due to the motion (described by the factor γ_n in (15)) and thus, we have to multiply the Huff's data by the factor b_n only. For the $1S$ state of the bound muon, $b_1 = 1-(Z\alpha)^2$ (10a). Hence we obtain the relationship

$$\tau_b = (1-(Z\alpha)^2)\tau_{\text{Huff}}, \tag{19}$$

where τ_b is the decay rate of the bound muon in PBFT, determined on the basis of (15), and the decay rate τ_{Huff} has been calculated by Huff after the accounting of all his corrections (the circles in Fig. 1). The half lives τ_b calculated via (19) are shown in Fig. 1 as triangles, and they lie within the range of standard deviation for all experimental points presented in Fig. 1 and thus provide a quantitative agreement with the experimental data of [10].

Therefore, we come to conclude that the experimental data [10] fully confirm the validity of (15) in its application to quantum bound systems. At the same time, one can add that in the later experiment [11] the data obtained in [10] by Yovanovitch at $Z=20\dots30$ and $Z>50$ were not reproduced. Thus, new careful experiments for verification of the original results of [10] are highly required for further test of the effect of variation of time rate for bound charged particles versus the electric potential, predicted within its framework of PBFT.

4 Conclusion

In the present paper we recalled the basic ideas of pure bound field theory (PBFT), we had previously developed [1], which implies the replacements $m \to mb_n$, $e\varphi \to \gamma_n e\varphi$ in the

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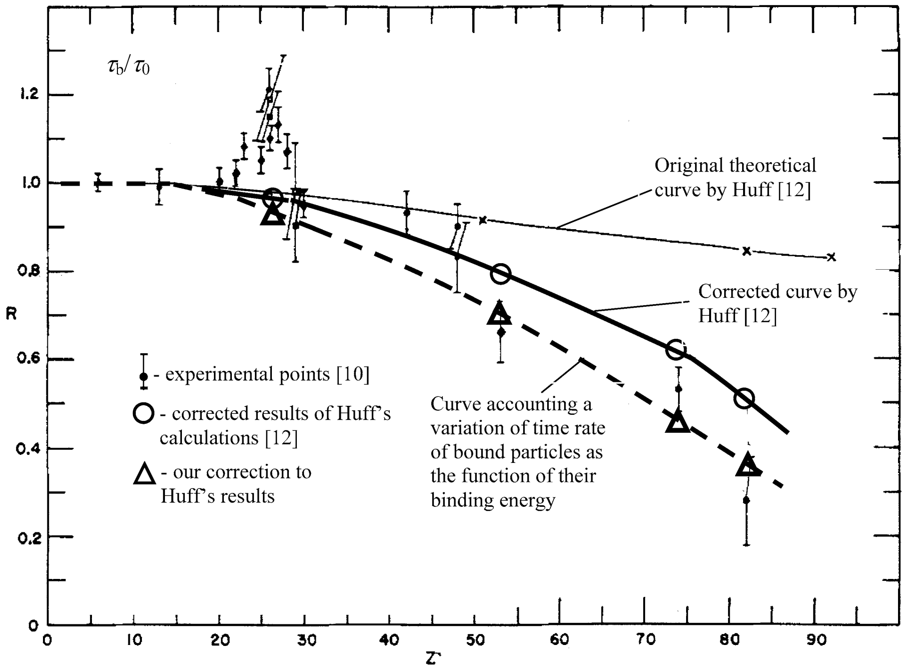

Fig. 1 Comparison of the results of theoretical calculation of the bound muon decay rate versus Z with the experimental data of [10]. The peak near $Z = 26$ observed in [10] was not confirmed in the later experiment [11]

Dirac equation for the bound electron in an external EM field. The introduction of these replacements into Dirac-Coulomb equation for the quantum one-body problem gives the same gross, as well as fine structure atomic energy levels, as those furnished by the common approach, but implies at the same time a scaling transformation $\mathbf{r} = \mathbf{r}'/(b_n\gamma_n)$. Analyzing this transformation in the classical limit of the one-body problem, we suggest interpreting the radial coordinates $\mathbf{r}$ and $\mathbf{r}'$ as belonging to different Lorentz frames attached to the resting nucleus and the orbiting electron, correspondingly. This interpretation leads to the change of the time rate for the orbiting electron $dt_e = b_n\gamma_n dt$ with its classical limit (16). In the framework of PBFT, such a change of electron's time rate occurs in a full conformity with the relativistic transformation of the quantities "time", "mass" and "total energy" and from the physical viewpoint frames a new physical phenomenon: more specifically this is the dependence of time rate for quantum bound charge on the electric potential of an external EM field. The experimental data obtained by Yovanovitch [10] on the decay rate of bound muon in meso-atoms fully support our approach leading to the disclosure of the mentioned new phenomenon.

Finally, it would be fair to mention that the present work was stimulated by the previous efforts of the first co-author with regards to the explanation of the decay of a bound muon [14–16], which led him to similar results, though through totally different means, which lie outside of this paper.

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